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Burning of char/carbon particle in a periodic thermal environment

K. ANNAMALAI and S. STOCKBRIDGE

Department of Mechanical Engineering, Texas A and M University, College Station, TX 77843, U.S.A.

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NOMENCLATURE

A_H	heat transfer area
B	pre-exponential factor
C_p	specific heat of gas
D	diffusion coefficient
d_p	diameter of particle at any time
E	activation energy
h	heat transfer coefficient
h_m	mass transfer coefficient
M	mass
$\dot{M}$	mass rate
$\dot{M}_p$	mass rate of contact of solids with heat exchangers
n	see equation (5)
Nu	Nusselt number
Pr	Prandtl number
Re	Reynolds number
Sh	Sherwood number
T	temperature
ΔT_{BH}	temperature difference between bed and heat exchanger
ΔT_p	particle temperature drop near heat exchanger
t	time
t_p	period of time
t'_h	heating time available for each slug of mass in non-uniform temperature field in the free space
t_h	heating time available for char in uniform temperature field in the free space
W	weight
Y	mass fraction.

Greek symbols

δ	bubble void fraction
ν_s	stoichiometric coefficient for oxygen
ρ	density
σ	Stefan–Boltzmann constant.

Subscripts

b	burning
c	cooling or contact
ch	char
h	heating
nc	no contact
o	oxygen
p	particle
p, 0	particle at time $t = 0$
R	radiation
s	solids (inert + char)
w	wall
∞	infinity.

1. INTRODUCTION

IN A FLUIDIZED bed combustor (FBC) fuel particles are introduced into a bed of sand (or dolomite and limestone) and burnt to release energy. A system of heat exchangers in the bed provides good control of the bed (or reaction zone) temperature within 1000–1200 K. As opposed to conventional combustion systems where heat exchangers are located away from the reaction zone and where the temperature of the reaction zone can be as high as 2000 K, the heat exchangers are located in the bed for a FBC. Thus thermal inhomogeneity is introduced to the circulating particles in a FBC.

The heat transport to the immersed tube surfaces of heat exchangers is explained by the packet renewal model [1–5]. Emulsions containing the solids at bulk temperature are transported to the heat exchangers due to the circulation pattern of the solids set up by rising bubbles. The particles undergo unsteady heat transfer to the tube surfaces if the characteristic time for cooling of the particles is comparable to residence time (~ 0.5–1 s [3–5]) on tube surfaces. Thus the heat transfer coefficient will be very large initially and will taper off as time progresses. Thus, the average heat transfer coefficient is extremely high. At the same time, inert sand particles along with char/carbon cool off rapidly, possibly to the tube wall temperature. Since contacts are made and broken by the rising bubbles, the emulsion will be alternatively drawn to the free space between the heat exchangers and to the tube surfaces and hence char particles can be subjected to an environment of alternative cooling and heating. This hypothesis seems to be confirmed by past experimental findings on heat transfer coefficients by Vakrushev *et al.* [6] who found that the heat transfer coefficient of a tube heat exchanger is higher than the value obtained for another tube located just immediately above it [pitch/dia = 4, dia 25 mm] for a fluidized bed fired with coke. This observation is attributed to cooling of particles past the lower tube. Many of the past models of combustion are based on the assumption of a bulk uniform temperature field around char particles [7–10]. Basu examined the effect of combustion of coal on the heat transfer rate to the immersed heat exchanger surfaces [11]. The present work was undertaken in order to investigate the effects of an alternating temperature field on the burning rate. It should be noted that the coal particles may see a fluctuating temperature field either due to the presence of heat exchangers or due to a turbulent thermal environment. For a turbulent intensity of 20% of the mean velocity (~ $1\ \mathrm{m\,s^{-1}}$) and for a distributor with nozzles of 0.20 cm in diameter, the time scale for turbulence is of the order of 10 ms. For the particle size under investigation (~1 mm diameter) the thermal inertia time is of the order of 1000 ms. Hence the effect of fluctuating temperature field due to turbulence on burning is neglected. Thus it is assumed that the oscillating temperature field as faced by the moving particles is entirely due to the presence of heat exchangers. For simplicity, single char particle combustion is analyzed first. As the char particle ascends through the FBC it is hypothesized as being exposed to a hot gas temperature (T_h) of 1100 K in the free spaces between the heat exchangers for a period of time t_h, and to a cold gas temperature (T_c) of 600 K on the heat exchangers for a period of t_c. Figure 1 shows the periodic thermal environment (T_∞ as a function of time) that a particle is exposed to as the particle moves through the FBC. When the particle is reacting, oxygen

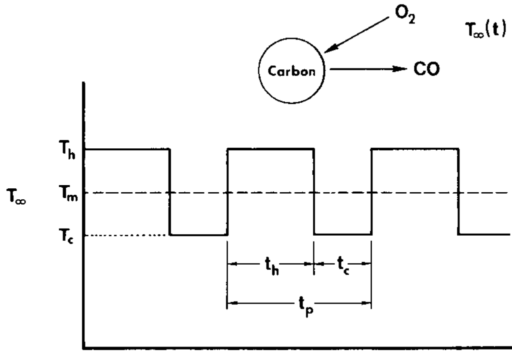

FIG. 1. Schematic diagram of the fluctuating temperature field surrounding a reacting carbon/char particle.

diffuses toward the surface and undergoes exothermic reaction producing CO. The thermal history and burning time of the particles are obtained as functions of t_h/t_c at a specified oxygen mass fraction in the free stream. The results reveal a significant increase in burning time and cooler particle temperature compared to bulk gas temperature as t_h/t_c is reduced. This will significantly affect the weight of char and hence the combustion efficiency.

2. GOVERNING EQUATIONS

The particle temperature history at any time t is given by the following differential equation (Fig. 1)

$$M_{ch}C_{ch}\frac{DT_p}{Dt} = \dot{M}_{ch}\Delta h_c - h(T_p - T_\infty)\pi d_p^2 - \varepsilon\sigma\pi d_p^2(T_p^4 - T_R^4). \tag{1}$$

The mass loss rate under the control of combined diffusion and reaction kinetics is given as

$$\dot{M}_{ch} = \pi d_p^2 B\rho Y_{o,w}\,e^{-E/R_uT_p} \tag{2}$$

where $Y_{o,w}$ is related by the following mass transfer equation:

$$\dot{M}_o = \dot{M}_{ch}v_s = h_m\pi d_p^2(Y_{o,\infty} - Y_{o,w})\rho D \tag{3}$$

$$\dot{M}_c = -\pi d_p^2\frac{Dd_p}{Dt}\rho_{ch}. \tag{4}$$

Within a given period of oscillation

$$\begin{aligned} T_\infty &= T_h, \quad nt_p < t < (nt_p + t_h), \quad n = 0,1,2,\ldots \\ &= T_c, \quad (nt_p + t_h) < t < (n+1)t_p, \quad n = 0,1,2,\ldots \end{aligned} \tag{5}$$

where n is the nth period of oscillation faced by the particle.

With the initial conditions at $t = 0$, $d_p = d_{p,0}$, $T_p = T_{p,0}$, equations (1) and (4) were integrated using the fourth-order Runge–Kutta integration scheme. Results for T_p and M_{ch} were then obtained as a function of time.

3. RESULTS AND DISCUSSIONS

Table 1(a) shows the data used for obtaining the results. The results are shown in Figs. 2 and 3 for particle temperature history and burning time vs t_h/t_c. Figure 2 shows the complete temperature history of the particles up to $t = 10$ s, and for $10 < t < 100$ s only the maximum and minimum temperatures attained by the particle are shown for clarity. For $t_h/t_c \sim 1$, the maximum temperature reached is less than the bulk gas temperature and the average temperature of the particle is about 200 K less than the gas temperature.* When t_h/t_c is increased to 4.0, the weighted average temperature of the reacting 1 mm char particle is close to the bulk gas temperature; but the burning time increases by more than a factor of two. However, the inert particles may not reach such a high temperature. Thus at $t_h/t_c = 4.0$ reacting char particles will supply heat not only to the gas at 1100 K but may supply heat to the inert particles which can be at temperatures less than 1100 K depending upon the thermal heat capacity of the inert particles. Figure 3 plots the burning time of char particles vs t_h/t_c for two Nusselt numbers at heat exchangers: (1) $Nu_c = 2$; (2) $Nu_c = 10$. The second case where $Nu_c = 10$ represents the number resulting from the use of a realistic bed to wall heat transfer model (see the footnote of Table 1). It is found that when $t_h/t_c \to \infty$ (i.e. no cooling) $t_b \sim 480$ s (the burning time checks with experimental results for 1 mm particles within a factor of two [8]) and the burning time exponentially increases as t_h/t_c is reduced (or as t_c/t_h is

* For low melting ash fuels, cooler particles can minimize the problem of agglomeration in the bed particularly when the bed is fired with biomass [13].

Table 1. Data used in calculations

(a) Combustion calculations (Reaction $C+\frac{1}{2}O_2 \rightarrow CO$)

B	$= 5\times10^7\,\mathrm{m\,s^{-1}}$ [12]
C_{ch}	$= 1414\,\mathrm{J\,kg^{-1}}$
C_p	$= 1175\,\mathrm{J\,kg^{-1}\,K^{-1}}$
D	$= 1\times10^{-4}\,\mathrm{m^2\,s^{-1}}$
$d_{p,0}$	$= 1$ mm
E	$= 150\,\mathrm{kJ\,mol^{-1}}$ [12]
Δh_c	$= 9203\,\mathrm{kJ\,kg^{-1}}$
Nu_h	$= 2$*, $Nu_c^* = 2, 10$
Sh	$= 2$*
T_c	$= 600$ K
T_h	$= 1100$ K
ρ	$= 0.25\,\mathrm{kg\,m^{-3}}$
ρ_{ch}	$= 800\,\mathrm{kg\,m^{-3}}$
ε	$= 0.8$
ν_s	$= 1.33$

(b) Heating time (t_h)/cooling time (t_c) calculations

A_h	$= 1.1\,\mathrm{m^2}$
Bed dimension: $0.46\times0.46\times0.4$ m [15]	
$C_{p,s}$	$= 1000\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Δh	$= 340\,\mathrm{W\,m^{-2}\,K^{-1}}$ (heat transfer coefficient)
ΔT_{BH}	$= 500$ K
ΔT_p	~ 100 K
t_c	$= 1/2$ s†
W	$= 50$ kg

* Actually in the free space $Nu = (2+0.6Re^{1/2})\varepsilon_v$. For $\varepsilon_v = 0.5$, $Re \sim 8$, $Nu_h \sim 2.0$. Near the cold heat exchangers, Glicksman and Decker [14] give an expression for the average Nusselt number as $Nu_c = (1-\delta)\,[9.42+0.42Re\,Pr]$. With $\delta \sim 0.1$–0.5, $Pr = 0.7$, Nu_c ranges from 9 to 5. For the Sherwood numbers, $Sh \cong Nu$ in free space while the results for burning at cold heat exchangers are expected to be almost independent of assumption for values of Sh since reaction is almost frozen at the heat exchangers.

† As a check on this value for the bed selected, the following calculation was carried out. For a monolayer of one particle diameter from the tube surface, and with a tube diameter of 0.0254 m, length 14.6 m, the amount of contact mass was calculated; thus $t_c \sim M_p/\dot{M}_p \sim 0.51$ s.

increased). For case (1), the plot of burning time vs t_c/t_h on semilog paper appears to be almost linear. The increased burning time results in increased char weight of the bed which results in more elutriation loss [9] with a consequent decrease in the combustion efficiency of the char. If char burning is modeled as combustion in uniform bulk temperature based on burning time calculations then t_h/t_c should be greater than 30

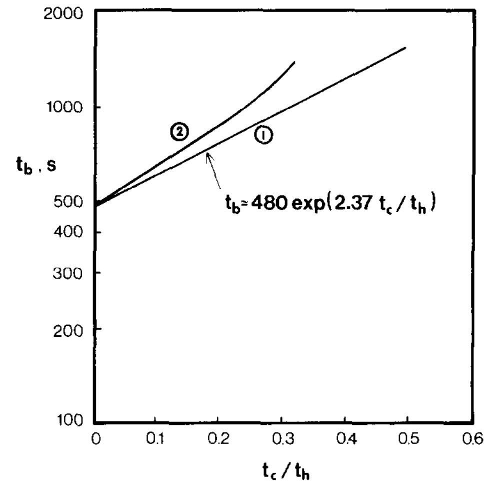

FIG. 3. Burning time (for 90% mass) vs ratio of contact time to heating time ($d_p = 1$ mm, $T_h = 1100$ K, $T_c = 600$ K): (1) $Nu_c = 2.0$; (2) $Nu_c = 10.0$.

for $d_p = 1$ mm, $T_h = 1100$ K and $T_c = 600$ K for an error within 5% of the results for combustion of char in a non-uniform thermal field.

The relationship between t_h/t_c and heat exchanger design will be discussed qualitatively. If the sensible heat lost by all contacting particles is equal to the augmentation in heat transfer rate to the in bed heat exchangers, then the required particle contact rate is given as

$$\dot{M}_p = \frac{\Delta h A_H \Delta T_{BH}}{C_{p,s}\Delta T_p}. \tag{6}$$

If particle contact time is t_c and noncontact time is t_{nc}, then the mass required at the exchanger surface during the period of contact is

$$M_p = \dot{M}_p(t_c+t_{nc}). \tag{7}$$

This mass M_p is normally less than the mass of the total material in the bed. If the bed mass of particles is W, and this is divided into slugs of M_p, then the initial slug of mass M_p which originally contacted the exchanger surface will contact again (on average) after a period of t_h' such that

$$t_h' = \left(\frac{W}{M_p}\right)(t_{nc}+t_c) \tag{8}$$

where $(t_{nc}+t_c)$ is a period of time available for each slug of mass to heat up every time other slugs contact the exchanger surface.* It was also assumed that every slug of mass has an

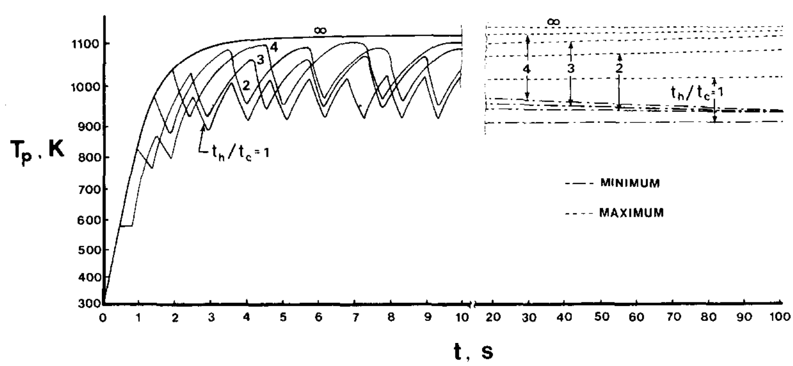

FIG. 2. Thermal history of a carbon/char particle ($d_p = 1$ mm, $T_h = 1100$ K, $T_c = 600$ K, $Nu_c = 2.0$).

equal probability of contacting the heat exchangers. Using equations (6) and (7) in equation (8)

$$\frac{t'_h}{t_c} = \left(\frac{WC_{p,s}\Delta T_p}{\Delta h A_H \Delta T_{BH} t_c}\right). \quad (9)$$

There is a difference between t'_h and t_h. It is possible that each slug of mass containing both inert particles and char may see a free space temperature less than the bulk temperature of the bed during the heating time if tubes are clustered (or if poor mixing exists), while the t_h we imposed on the char particle assumes a temperature in the free space equal to the bulk temperature. Thus invariably t'_h/t_c should be set such that t'_h/t_c is always higher than the required t_h/t_c which will give a burning time closer to the time scales obtained in the absence of heat exchangers. Otherwise the bed weight will increase with a consequent increase in elutriation loss and a decrease in combustion efficiency. An estimation can be made for t'_h/t_c. Table 1(b) lists the data used in the calculations. Assuming that char is distributed uniformly in the circulating solids $(t'_h/t_c)_{char} \sim 50$ which is double than required t_h/t_c. However, if W/A_H is less (less mass and more heat transfer area), then t'_h/t_c will decrease. If the ratio W/A_h is such that $t'_h/t_c < 30$, then one must account for the effect of periodic thermal environment on the burning rate of char particles.

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* It is possible that a fraction of the mass of M_p may contact the heat exchangers within a period of time $t < t'_h$ and another fraction may contact the tubes within $t > t'_h$. This t'_h is presumed to be the average period of heating time available for the slug.